TAKING ACCOUNT OF THE NONUNIFORMITY OF THE DYNAMICAL LOADING OF A SAMPLE IN EXPERIMENTS ON POLARIZATION OF DIELECTRICS UNDER SHOCK COMPRESSION

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Polarization in shockwaves (shock polarization henceforth) is understood to be the origination of a dipole moment in the bulk of a dielectric under the effect of a shock.

Experiments on shock polarization afford the possibility of obtaining information about a whole series of physical properties of the shock-compressed substances. Moreover, the interest in studying this phenomenon is governed by the possibility, in principle, of clarifying the character of nonequilibrium processes occurring in the shock transition and of differentiating between dynamic and static compression.

The phenomenological theories of shock polarization developed in [1-6] permit computation of the time dependence of the polarization current in the uniform case, by means of the magnitudes of the compression δ of the material, the dielectric permittivity of the initial material ε_1, the initial polarization of the material in the shock front P_0, as well as the dielectric permittivity ε_2, electrical conductivity ρ, and time of mechanical relaxation of the polarization τ of the material behind the shock front. However, because of imperfections of explosive generators of a plane shock, the dynamic loading of the sample being investigated is not strictly uniform in real tests. As is shown herein, taking account of this circumstance is necessary in many cases for a correct interpretation of experimental oscillograms within the scope of existing theories.

Let us note that modern high-speed oscillographs have a build-up time not exceeding several nsec, which is considerably less than the magnitudes of the time-difference, realizable in experiments, of the shock entrance into samples. Hence, the need to take account of distortion of the polarization signal because of the finite frequency band of the measuring channel does not ordinarily occur.

1. General Case of Deviation from the Uniform Character of Shock Stress

Analogously to the usual formulation of the experiments, [7, 8] for instance, let us consider the sample under investigation of thickness x_0 to be the dielectric of a flat condenser formed by a metal screen and electrode. In the x, y, z coordinate system the screen through which the shock enters the sample is located in the x = 0 plane. Polarization in the initially isotropic dielectric originates on any infinitesimal portion of the shock front surface along the isolated direction given by the velocity vector of the shock front **U**.

In a first approximation (without taking account of the curvature of the lines of force of the electrical field), we obtain an expression for the polarization current j(t) in the case of an arbitrary shock front surface by considering the direction of the vector **U**, meaning the polarization vector at each point of this surface close to the direction of the x axis, i.e.,

$$U_x \approx U \tag{1.1}$$

where U_x is the projection of **U** on the x axis and $U = |\mathbf{U}|$.

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In the one-dimensional case, the current in the short-circuit loop of polarization sensor can be represented as [1-6]

$$i(t) = S_0 f(t), \qquad 0 \leqslant t \leqslant t_1 = x_0/U \tag{1.2}$$

Here S_0 is the electrode area, and t_1 is the time of shock passage in the sample.

Taking account of (1.1) for a shock front of arbitrary shape which starts to enter the sample at t = 0, it follows from (1.2) that the contribution $\Delta j(t)$ to the total current from the shock loaded part of the dielectric of area $\Delta y \Delta z$ with coordinates y, z equals

$$\Delta j(t) = f(x(y, z, t)\, U^{-1})\, \Delta y \Delta z \tag{1.3}$$

Passing over to differentials in (1.3) and integrating over the shock loaded part of the sample area,* we obtain for the actual polarization current

$$j(t) = \iint f\left(\frac{x(y, z, t)}{U}\right) dy dz \tag{1.4}$$

2. Distortion of the Shock Front Shape as Curvature

The main kind of distortion of the shock front shape is the curvature for sufficiently small sample areas in the axially symmetric explosive systems ordinarily used. A shock front in the form of an almost ideal spherical surface of practically arbitrary radius can be obtained comparatively simply, especially in experiments using spherical detonation. Hence, let us examine this important particular case of nonuniform dynamic loading in more detail as applied to shock polarization experiments.

For definiteness, let us consider the dielectric sample to have the shape of a cylinder with base radius r_0, where

$$r_0 \gg x_0 \tag{2.1}$$

Let us represent the shock front as a part of the spherical surface with center at the point $x = -R_0$, $y = 0$, $z = 0$ and radius $R = R_0 + Ut$. For this case condition (1.1) is equivalent to the condition

$$R_0 \gg r_0 \tag{2.2}$$

It is convenient to conduct the further examination in the cylindrical coordinates

$$y = r\cos\varphi, \qquad z = r\sin\varphi, \qquad x = x$$

The equation of the shock front surface is written as

$$x^2 + 2R_0 x + r^2 - 2R_0 Ut - U^2 t^2 = 0 \tag{2.3}$$

Taking (2.1) and (2.2) into account, and also that

$$x^2 \leqslant U^2 t^2 \leqslant x_0^2 \tag{2.4}$$

we neglect the terms x^2 and U^2t^2 in (2.3), after which we obtain

$$x/U = t - r^2/2R_0 U \tag{2.5}$$

It follows from (2.5) with x = 0 that the area of the shock loaded part of the sample base $S = \pi r^2$ changes according to a linear law

$$S = 2\pi R_0 U t \tag{2.6}$$

up to the time t_2 of total loading of the whole base area. Then this area remains equal to S_0.

The quantity t_2 is the difference in time of shock entry into the sample under investigation and can be found from (2.6) for $S = S_0$:

$$t_2 = S_0 / 2\pi R_0 U \tag{2.7}$$

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*Without limiting the generality, we shall henceforth consider the electrode to cover the whole sample area.

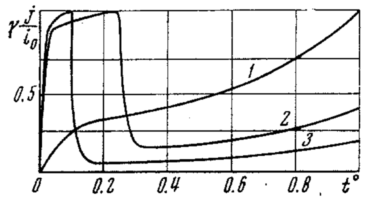

Fig. 1

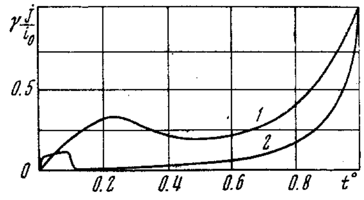

Fig. 2

Passing to cylindrical coordinates in (1.4), substituting (2.5) into the expression obtained, and integrating with respect to the angle between 0 and 2π, we have

$$j(t) = 2\pi \int_0^r f\left(t - \frac{\eta^2}{2R_0U}\right)\eta d\eta \tag{2.8}$$

where from (2.6) and (2.7)

$$r = \begin{cases} \sqrt{2R_0Ut}, & t \leqslant t_2 \\ \sqrt{2R_0Ut_2}, & t \geqslant t_2 \end{cases} \tag{2.9}$$

By using the change of variable $\eta^2/2R_0U = \xi$ and taking account of (2.7), we finally obtain from (2.8) and (2.9)

$$j(t) = \begin{cases} \frac{S_0}{t_2}\int_0^t f(t-\xi)\,d\xi, & t \leqslant t_2 \\ \frac{S_0}{t_2}\int_0^{t_2} f(t-\xi)\,d\xi, & t \geqslant t_2 \end{cases} \tag{2.10}$$

The relationship (2.10) was integrated numerically on an electronic computer in order to clarify the character of the influence of shock front curvature on the time dependence of the polarization current. Hence, a sufficiently general expression for i(t) obtained in [1] and including the solutions found in [2, 4] as particular cases:

$$i(t) = \frac{i_0 \varkappa t_1}{\psi(t)\exp(t/\tau)}\left\{1 - \frac{\varkappa(t_1 - t) + \theta(1-\varkappa)}{\theta[\psi(t)]^{\varphi}}\exp\left(\frac{t}{\mu}\right)\int_0^t [\psi(y)]^{\varphi-1}\exp\left(-\frac{y}{\mu}\right)dy\right\} \tag{2.11}$$

$$(\varkappa = \varepsilon_2\delta/\varepsilon_1, \quad i_0 = P_0S_0/\varkappa t_1, \quad \theta = \rho\varepsilon_2/4\pi, \quad \psi(t) = \varkappa t_1 + (1-\varkappa)t,$$
$$\varphi = 1 + \varkappa t_1/\theta(1-\varkappa)^2,$$
$$\mu = \tau\theta(1-\varkappa)/\theta + \varkappa(\tau - \theta))$$

was used as $S_0 f(t)$.

The assumption about short-circuiting the equivalent polarization generator for this case corresponds to the fact that the time constant of the measurement circuit is much less than t_1, τ, and $\rho\varepsilon_2/4\pi$.

Some characteristic computational time dependences of the polarization current are presented in Figs. 1-3. The dimensionless time $t° = t/t_1$ and the parameters $\theta° = \theta/t_1$, $\tau° = \tau/t_1$, $t_2° = t_2/t_1$ were used in construction of these curves, and values of the function $j(t°)/i_0$ are multiplied by the scale factor γ. The quantity γ was selected in such a manner that the maximum value of $\gamma j(t°)/i_0$ on the segment $0 \leq t° \leq 1$ was one.

The theoretical dependences $j(t°)$ in Fig. 1 correspond to the following cases: curve 1 to $\varkappa = 2.0$, $\tau° = 50$, $\theta° = 50$, $t_2° = 0.1$, $\gamma = 0.28$; curve 2 to $\varkappa = 2.1$, $\tau° = 0.01$, $\theta° = 50$, $t_2° = 0.25$, $\gamma = 22$; and curve 3 to $\varkappa = 2.1$, $\tau° = 0.01$, $\theta° = 50$, $t_2° = 0.1$, $\gamma = 9.52$.

The theoretical dependences $j(t°)$ are represented in Fig. 2 by curve 1 with $\varkappa = 2.0$, $\tau° = 50$, $\theta° = 0.1$, $t_2° = 0.25$, $\gamma = 0.74$; and curve 2 with $\varkappa = 2.0$, $\tau° = 50$, $\theta° = 0.01$, $t_2° = 0.1$, $\gamma = 0.96$.

The theoretical dependences $j(t°)$ in Fig. 3 are curve 1 with $\varkappa = 2.1$, $\tau° = 0.1$, $\theta° = 0.01$, $t_2° = 0.25$, $\gamma = 32$; and curve 2 with $\varkappa = 2.0$, $\tau° = 0.1$, $\theta° = 0.1$, $t_2° = 0.1$, $\gamma = 2.50$.

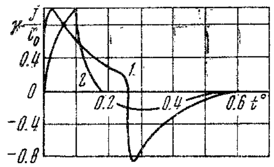

Fig. 3

Let ourselves be limited to the case $\varkappa \geq 1$ apparently realizable in the majority of experiments, and let us note some peculiarities of the computed curves of $j(t^\circ)$.

For τ°, $\theta^\circ > 1$ (curve 1 in Fig. 1, for instance) the values of the function $j(t^\circ)$ for $t^\circ > t_2^\circ$ differ comparatively little from the corresponding values of the function $i(t^\circ)$. The initial current i_0 in the experiments can be determined sufficiently accurately by extrapolating $j(t^\circ)$ to $t^\circ = 0$. Other conditions being equal, an increase in t_2° results in a diminution of the ratio between the final current $j_1 = j(1)$ and the initial current i_0.

If $\tau^\circ \ll 1$, but $\theta^\circ > 1$ (curves 2 and 3 in Fig. 1), or $\tau^\circ > 1$, but $\theta^\circ \ll 1$ (Fig. 2), the computational dependences $j(t^\circ)$ have a maximum j_2 for $t^\circ = t_2^\circ$.

In the case under consideration the shock front curvature results in a radical difference between the polarization curves $j(t^\circ)$ and those computed by using (2.11). In particular, as is seen graphically from Fig. 1 (curves 2 and 3), for example, as t_2° increases the ratios i_0/j_2 (γ increases) and j_1/j_2 increase.

Let us also note that for reasonable values of $\varkappa$, if $\theta^\circ > 1$ but $\tau^\circ \ll 1$ then $j_1 < j_2$ holds and if $\theta^\circ \ll 1$ but $\tau^\circ > 1$ then $j_1 \gg j_2$.

The distinctive peculiarity of the depencence $j(t^\circ)$ for τ°, $\theta^\circ \ll 1$ (Fig. 3) is the change in sign of the polarization signal near the point t_2°.

Now, let us examine the important particular case of low amplitude shocks; i.e., let us consider that $\varkappa = 1$ and $\theta^\circ = \infty$. In this approximation we easily obtain from (2.11)

$$i(t^\circ) = i_0 \exp(-t^\circ/\tau^\circ) \tag{2.12}$$

Hence, applying (2.10) we have for the current $j(t^\circ)$

$$j(t^\circ) = \begin{cases} \dfrac{i_0\tau^\circ}{t_2^\circ}\left[1 - \exp\left(\dfrac{-t^\circ}{\tau^\circ}\right)\right] & (t^\circ \leqslant t_2^\circ) \\ \dfrac{i_0\tau^\circ}{t_2^\circ}\left[\exp\left(\dfrac{t_2^\circ}{\tau^\circ}\right) - 1\right]\exp\left(-\dfrac{t^\circ}{\tau^\circ}\right) & (t^\circ \geqslant t_2^\circ) \end{cases} \tag{2.13}$$

It follows from this relationship that for sufficiently small τ° ($\tau^\circ \ll t_2^\circ$) the maximum amplitude of the initial current j_2 and its duration, which equals t_2° approximately, depend essentially on the difference in time of shock entry into the samples under investigation. In particular, we obtain for j_2

$$j_2 \approx i_0\tau^\circ/t_2^\circ \tag{2.14}$$

This expression permits an approximate estimation of the quantity τ° or i_0, respectively, by measuring j_2 and t_2° experimentally for a known value of i_0 or τ°.

An analytical expression analogous to (2.14) for j_2 can also be obtained in the case $\tau^\circ = \infty$, $\varkappa = 1$, $\theta^\circ \ll t_2^\circ$. Indeed, neglecting second order terms in the expression for $i(t^\circ)$ from [4] for $t^\circ \ll 1$, and evaluating the integral, we have

$$i(t^\circ) = i_0 \exp(-t^\circ/\theta^\circ) \tag{2.15}$$

Using this formula exactly as (2.14) was obtained from (2.12), it is easy to show that for $\theta^\circ \ll t_2^\circ$

$$j_2 \approx i_0\theta^\circ/t_2^\circ \tag{2.16}$$

Numerical computations on an electronic computer by means of (2.10) in the ranges of parameter variation $0.2 \leq \varkappa \leq 2.1$, $5 \cdot 10^{-4} \leq \tau^\circ \leq 10^{-2}$, $5 \cdot 10^{-4} \leq \theta^\circ \leq 10^{-2}$, and $0.05 \leq t_2^\circ \leq 0.25$ permitted it to be found that the relationships (2.14) and (2.16) obtained for $\varkappa = 1$ are satisfied with a good degree of accuracy ($\sim 30\%$) for quantities $\varkappa$ in, at least, the range $0.2 \leq \varkappa \leq 2.1$.

Experimental material existing at this time in the case of the presence of one shock polarization mechanism is described satisfactorily by the relationships (2.10) and (2.11).

Dependences $j(t^\circ)$ of the type of curve 1 in Fig. 1 are observed in specific dynamic pressure domains for many organic polymers [7, 9] and a number of ionic crystals [8, 10, 11].

Dependences j(t°) analogous to curves 2 and 3 in Fig. 1 are characteristic for low-molecular-weight fluid dielectrics [12] at pressures behind the shock front which are not sufficient for the origination of substantial electrical conductivity.

Polarization signals of the form of Fig. 2 are determined in tests with water [9, 13], Plexiglas and polystyrene at pressures above the phase transition [7], vinyl plastic [14], low-molecular-weight benzene derivatives [12], trinitrotoluene [15], etc. However, the interpretation of such oscillograms must be approached with care since the same experimental dependences j(t°) can be related not to the case $\theta^\circ \ll 1$, $\tau^\circ > 1$ within the scope of (2.10) and (2.11), but to the presence of a transition electrical conductivity zone with $\tau^\circ \ll 1$ behind the shock front.

3. Simple Model of a Transition Electrical Conductivity Zone behind a Shock Front

It is known that many dielectrics become ionic conductors under a sufficient dynamic pressure magnitude [16]. Since the heterolytic dissociation of molecules of the initial material,which is apparently thermal in nature and being relieved because of ion association at high pressure, should occur in a finite time, the appearance of a transition electrical conductivity zone should be expected behind the shock front. In the simplest case the electrical conductivity can originate some time after passage of the shock front. Transition electrical conductivity zones have been observed experimentally in explosive materials [15, 17] and nitrobenzene [12].

Let us examine an approximate electrodynamic model which will permit obtaining a time dependence of the polarization current by taking account of the shock front curvature upon the origination of electrical conductivity with a delay greater than τ.

Let us consider the electrical conductivity to grow to infinity by a jump within a distance Δ behind the shock front, where the quantity Δ satisfies the relation

$$\tau \ll \Delta/(U-u) \qquad (3.1)$$

Under this condition the mechanical relaxation process will occur in the dielectric medium and will be completed in practice upon the origination of electrical conductivity. Hence, for $t < \Delta/(U-u)$ the dependence j(t) can be obtained by using (2.10) on the expression for i(t) obtained in [2]. The qualitative form of this dependence is analogous to curves 2 and 3 in Fig. 1. For $t = \Delta/(U-u)$ a current jump occurs associated with the initial condition of the jump in electrical conductivity.

Now, let us find the dependence j(t) for $t > \Delta/(U-u)$. In the one-dimensional case let us consider the motion of a shock front through a plane-parallel condenser with the material under investigation. Let us assume that all the vectors are perpendicular to the plane of the shock front, the front carries no free charges, and the mechanical relaxation is described by the expression

$$P(x, t) = P_0 \exp\left(-\frac{Ut-x}{\tau(U-u)}\right) \qquad (3.2)$$

Here P(x, t) is the dipole moment per unit volume of material induced by the shock, P_0 is the initial polarization on the shock front, and $(Ut-x)/(U-u)$ is the time that a volume of material with coordinate x is in the shock-compressed state.

From the continuity condition of the electrical bias D upon crossing the shock front we obtain for a plane-parallel condenser

$$D = \frac{4\pi Q}{S_0} = \varepsilon_1 E_1 = \varepsilon_2 E_2 + 4\pi P(x, t) \qquad (3.3)$$

where Q is the charge on the condenser, E_1 and E_2 are the electrical field intensity in the initial and compressed dielectric, respectively.

The potential difference V(t) on the condenser plates is

$$V(t) = \frac{4\pi Q}{S_0 \varepsilon_1}(x_0 - Ut) + \frac{4\pi Q}{S_0 \varepsilon_2}\Delta - \frac{4\pi}{\varepsilon_2}\int\limits_{Vt-\Delta}^{Vt} P(x, t)\, dx \qquad (3.4)$$

Let us limit ourselves to the short-circuit mode of the equivalent generator V(t) ≡ 0. Substituting (3.2) into (3.4) and evaluating the integral we obtain

$$Q = \frac{P_0 S_0 \varepsilon_1 \tau (U-u)}{\varepsilon_2 (x_0 - Ut) + \varepsilon_1 \Delta} \left[1 - \exp\left(-\frac{\Delta}{(U-u)\tau}\right)\right] \quad (3.5)$$

Taking account of (3.1), the exponential in this expression can be neglected as compared with one. Furthermore, differentiating the charge with respect to the time, introducing $\varkappa$, t°, τ°, and $t_3^\circ = \Delta/(U-u)t_1$, we find the polarization current

$$i(t^\circ) = \frac{P_0 S_0 \varkappa \tau^\circ}{t_1 [\varkappa (1 - t^\circ) + t_3^\circ]^2} \quad (3.6)$$

The time dependence of the polarization current taking account of the shock front curvature can be found by applying (2.10) to this expression. In particular, for $t^\circ \geq t_2^\circ$ we have

$$j(t^\circ) = \frac{P_0 S_0 \varkappa \tau^\circ}{t_1 [\varkappa (1 - t^\circ + t_2^\circ) + t_3^\circ] [\varkappa (1 - t^\circ) + t_3^\circ]} \qquad (t_3^\circ \gg \tau^\circ) \quad (3.7)$$

The polarization curves computed by means of (3.7) agree qualitatively with the dependences $j(t^\circ)$ presented in Fig. 2.

Therefore, for sufficiently small τ° the appearance of a transition electrical conductivity zone behind the shock front should result with a qualitative change in the dependence $j(t^\circ)$ being expressed in the transition of the polarization curves of the form in Fig. 3 to curves of the form in Fig. 2. The fact that dependences $j(t^\circ)$ of the form in Fig. 3 have not been observed experimentally up to now can be related to the generality of this phenomenon.

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